Cognition and conditionals

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in collaboration with

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Outline

- Conditionals in psychology
  - Indicative conditionals
  - Uncertain conditionals
- Mental probability logic
  - Wasons selection task
  - Truth table task
  - Paradoxes of the material conditional
- Conclusions

Conditionals in psychology: Indicative conditionals

Three prominent psychological predictions of how people interpret "If A, then B":

- Material conditional, $A \supset B$
- Conjunction, $A \wedge B$
- Conditional event, $B|A$

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Three prominent psychological predictions of how people interpret "If A, then B":

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	A	B	*Material conditional* $A \supset B$	*Conjunction* $A \wedge B$	*Conditional event* $B|A$
s_1	*true*	*true*	*true*	*true*	*true*
s_2	*true*	*false*	*false*	*false*	*false*
s_3	*false*	*true*	*true*	*false*	undetermined
s_4	*false*	*false*	*true*	*false*	undetermined

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s_3	*false*	*true*	*true*	*false*	undetermined
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Conditionals in psychology: Uncertain conditionals

$P(A \supset B)$

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Probabilistic extension
of the *mental model* theory

Johnson-Laird et al.

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Theoretical problems:

Paradoxes of the material conditional:

e.g., from $\boxed{B}$ infer $\boxed{\textbf{if } A\textbf{, then } B}$

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The material conditional is not a genuine conditional

$$(A \supset B) \quad \Leftrightarrow \quad (\neg A \vee B)$$

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Theoretical problems solved:

No paradoxes of the material conditional:

From $\boxed{P(B) = x}$ infer $\boxed{P(B|A) \in [0, 1]}$

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But: from $\boxed{P(B) = x}$ infer $\boxed{P(A \supset B) \in [x,1]}$

The conditional event $B|A$ is a genuine conditional!

Conditionals in psychology: Uncertain conditionals

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$P(B|A)$

Empirical Result:

$P(B|A)$ best predictor
for "**if** A**, then** B"

Evans, Over et al.

Oberauer et al.

Liu

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Probabilistic relation between
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- $P(A \supset B)$, $P(A \wedge B)$
  - Probabilistic extension of the *mental model* theory
    Johnson-Laird et al.
- $P(B|A)$
  - Probabilistic relation between premise(s) and conclusion
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  - Deductive relation between premise(s) and conclusion
    Mental probability logic
    Pfeifer & Kleiter
  - Empirical Result:
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Mental probability logic

- embedded in a probability logic framework

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  - $\overbrace{P(B|A) = x\,, \quad P(A) = y}^{premises} \models \overbrace{P(B) \in [xy, xy + 1 - y]}^{conclusion}$

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- coherence

Coherence

- de Finetti, and {Lad, Walley, Scozzafava, Coletti, Gilio,...}
- degrees of belief
- complete algebra is not required
- conditional probability, $P(B|A)$, is primitive
- zero probabilities are exploited to reduce the complexity
- imprecision
- provides semantics for System P

Wasons selection task

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If there is a vowel on the one side ,
then there is an even number on the other side .

Wasons selection task

If there is a vowel on the one side ,
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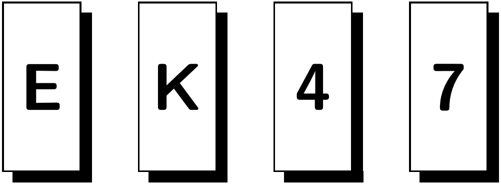
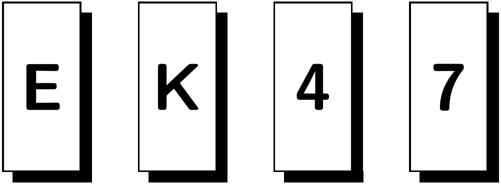


Wasons selection task

If there is a vowel on the one side (A),
then there is an even number on the other side (B).



A $\neg A$ B $\neg B$

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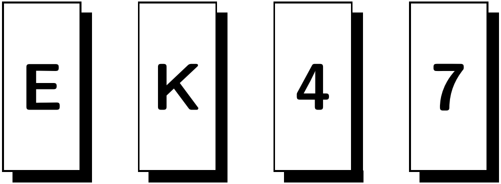

A $\neg A$ B $\neg B$

- 46% choose both, the A- and the B-card (Wason & Johnson-Laird, 1972)

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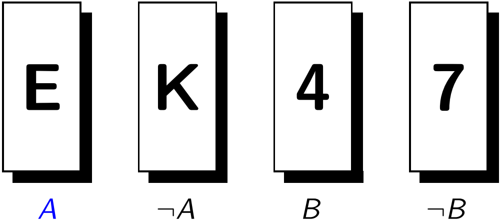

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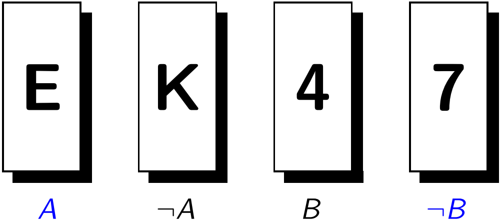

- 46% choose both, the A- and the B-card (Wason & Johnson-Laird, 1972)
- 33% choose the A-card (Wason & Johnson-Laird, 1972)
- 4% choose both, the A- and the $\neg B$-card (Wason & Johnson-Laird, 1972)

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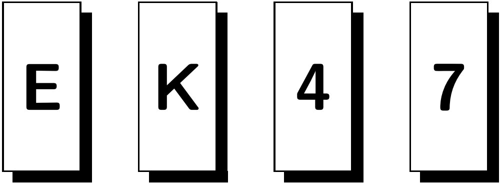

A	$\neg A$	B	$\neg B$
$A \supset B$	$A \supset B$	$A \supset B$	$A \supset B$
B	$\neg B$	A	$\neg A$
✓			✓
(MP)	(DA)	(AC)	(MT)

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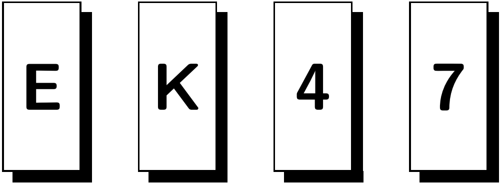

A	$\neg A$	B	$\neg B$
$A \equiv B$	$A \equiv B$	$A \equiv B$	$A \equiv B$
B	$\neg B$	A	$\neg A$
✓	✓	✓	✓

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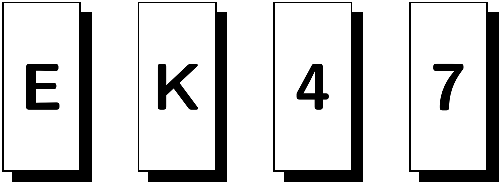

A	$\neg A$	B	$\neg B$
$A \wedge B$	$A \wedge B$	$A \wedge B$	$A \wedge B$
B	$\neg B$	A	$\neg A$
✓	✓	✓	✓

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Wasons selection task

If there is a vowel on the one side (A),
then there is an even number on the other side (B).

E	K	4	7
A	$\neg A$	B	$\neg B$
$A \wedge B$	$A \wedge B$	$A \wedge B$	$A \wedge B$
B	$\neg B$	A	$\neg A$
✓	✓	✓	✓

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Wasons selection task

If there is a vowel on the one side (A),
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E	K	4	7
$P(A) = 1$	$P(\neg A) = 1$	$P(B) = 1$	$P(\neg B) = 1$
$P(A \supset B) = 1$	$P(A \supset B) = 1$	$P(A \supset B) = 1$	$P(A \supset B) = 1$
$P(B) = 1$	$P(\neg B) \in [0, 1]$	$P(A) \in [0, 1]$	$P(\neg A) = 1$
✓			✓

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E	K	4	7
$P(A) = 1$	$P(\neg A) = 1$	$P(B) = 1$	$P(\neg B) = 1$
$P(B\|A) = 1$	$P(B\|A) = 1$	$P(B\|A) = 1$	$P(B\|A) = 1$
$P(B) = 1$	$P(\neg B) \in [0,1]$	$P(A) \in [0,1]$	$P(\neg A) = 1$
✓			✓

Wasons selection task

If there is a vowel on the one side (A),
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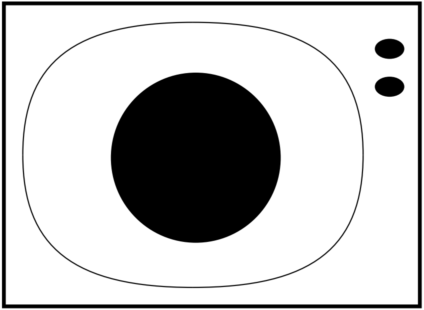
E	**K**	**4**	**7**
$P(A) = 1$	$P(\neg A) = 1$	$P(B) = 1$	$P(\neg B) = 1$
$P(A \wedge B) = 1$	$P(A \wedge B) = 1$	$P(A \wedge B) = 1$	$P(A \wedge B) = 1$
$P(B) = 1$	incoherent!	$P(A) = 1$	incoherent!
✓		✓	

Truth table task

Task AA, SP condition

If there is a circle on the screen, then the circle is black.

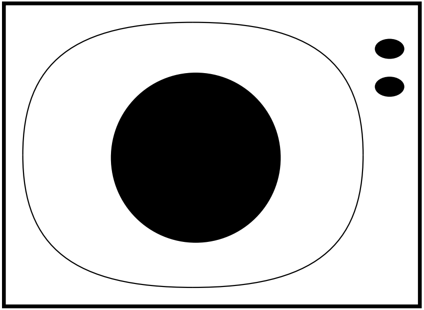


Does the shape on the screen speak for the assertion in the box?

☐ speaks against ☐ neither/nor ☐ speaks for

Task AA, PS condition

If there is a **black** shape on the screen, then it is a **circle**.



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Task AN, SP condition

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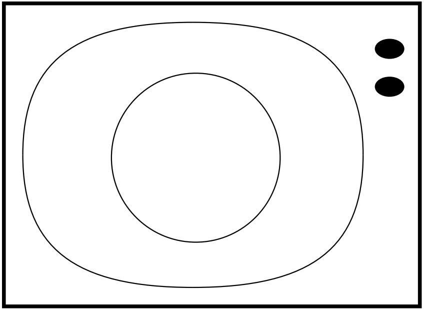

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Task NA, SP condition

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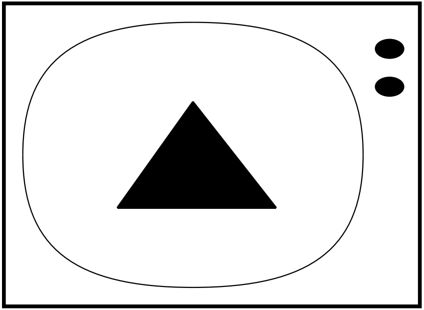

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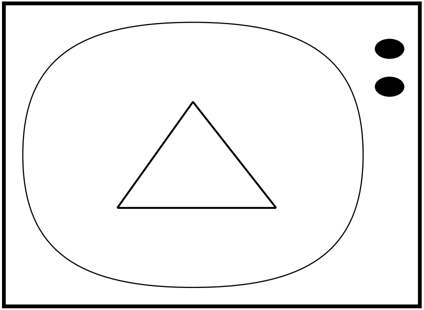
Task NN, SP condition

If there is a circle on the screen, then the circle is black.



Does the shape on the screen speak for the assertion in the box?

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Design

- Two conditions: SP ($n_1 = 18$) and PS ($n_2 = 18$)
- 16 target tasks: 4 conditionals $\times$ 4 truth table cases
- Order of tasks:

Conditional in box	Shape on screen	Task type
If circle, then black	●	target AA
If circle, then black	○	target AN
If circle, then black	▲	target NA
If circle, then black	△	target NN

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counterfactual	broken screen	filler item

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counterfactual	broken screen	filler item
If circle, then white	●	target AN
If circle, then white	○	target AA
If circle, then white	▲	target NN
If circle, then white	△	target NA
counterfactual	broken screen	filler item
⋮	⋮	⋮
If triangle, then white	△	target AA

Results: Mean Response Percentages

Group	Response	Task Type			
		AA	AN	NA	NN
SP	speaks against	2.78	86.11	30.56	22.22
	neither/nor	4.17	11.11	**61.11**	**76.39**
	speaks for	93.06	2.78	8.33	1.39

Representation as a conditional event $(\cdot|\cdot)$

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Representation as a conjunction $(\cdot \wedge \cdot)$

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Representation as a material conditional $(\cdot \supset \cdot)$

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Not yet clear what's going on here.

Paradoxes of the material conditional

Two paradoxes of the material conditional (conditional introduction):

"**If** A, **then** B" interpreted as "$A \supset B$"

$\mathfrak{P}$ $2 + 2 = 4$

---

log. valid

$\mathfrak{C}$ **If** the moon is made of green cheese, **then** $2 + 2 = 4$

$$\boxed{B} \vdash \boxed{A \supset B}$$

Two paradoxes of the material conditional (conditional introduction):

"**If** A, **then** B" interpreted as "$A \supset B$"

$\mathfrak{P}$ **Not**: The moon is made of green cheese

———————————————— log. valid

$\mathfrak{C}$ **If** the moon is made of green cheese, **then** $2 + 2 = 4$

$$\boxed{\neg A} \vdash \boxed{A \supset B}$$

Two paradoxes of the material conditional (conditional introduction):
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Mental model theory postulates that subjects represent "basic conditionals" "**If** A, **then** B" as

- implicit mental models:

A	B
$\ldots$	

…truth conditions of the conjunction, $A \wedge B$

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- implicit mental models
- explicit mental models:

A	B
not-A	B
not-A	not-B

...truth conditions of the material conditional, $A \supset B$

Two paradoxes of the material conditional, $A \supset B$

Example (Paradox 1)

$B \therefore$ If A, then B

Premise		*Conclusion*	
B	$\therefore$	$A \supset B$	(logically valid)
$P(B) = x$	$\therefore$	$P(A \supset B) \in [x, 1]$	(prob. informative)
$P(B) = x$	$\therefore$	$P(A \wedge B) \in [0, x]$	(prob. informative)
$P(B) = x$	$\therefore$	$P(B\|A) \in [0, 1]$	(prob. non-informative)

Two paradoxes of the material conditional, $A \supset B$

Example (Paradox 1)

B ∴ If A, then B

B ∴ If A, then not-B

Premise		*Conclusion*	
B	∴	$A \supset B$	(logically valid)
$P(B) = x$	∴	$P(A \supset B) \in [x, 1]$	(prob. informative)
$P(B) = x$	∴	$P(A \wedge B) \in [0, x]$	(prob. informative)
$P(B) = x$	∴	$P(B \vert A) \in [0, 1]$	(prob. non-informative)
B	∴	$A \supset \neg B$	(not logically valid)
$P(B) = x$	∴	$P(A \supset \neg B) \in [1-x, 1]$	(prob. informative)
$P(B) = x$	∴	$P(A \wedge \neg B) \in [0, 1-x]$	(prob. informative)
$P(B) = x$	∴	$P(\neg B \vert A) \in [0, 1]$	(prob. non-informative)

Two paradoxes of the material conditional, $A \supset B$

Example (Paradox 1)

$B \therefore$ If A, then B

$B \therefore$ If A, then not-B

Premise		*Conclusion*	
B	$\therefore$	$A \supset B$	(logically valid)
$P(B) = 1$	$\therefore$	$P(A \supset B) = 1$	(prob. informative)
$P(B) = 1$	$\therefore$	$P(A \wedge B) \in [0, 1]$	(pract. non-informative)
$P(B) = 1$	$\therefore$	$P(B\|A) \in [0, 1]$	(prob. non-informative)
B	$\therefore$	$A \supset \neg B$	(not logically valid)
$P(B) = 1$	$\therefore$	$P(A \supset \neg B) \in [0, 1]$	(pract. non-informative)
$P(B) = 1$	$\therefore$	$P(A \wedge \neg B) = 0$	(prob. informative)
$P(B) = 1$	$\therefore$	$P(\neg B\|A) \in [0, 1]$	(prob. non-informative)

Two paradoxes of the material conditional, $A \supset B$

Example (Paradox 2)

Not-A $\therefore$ If A, then B

Premise		*Conclusion*	
$\neg A$	$\therefore$	$A \supset B$	(logically valid)
$P(\neg A) = x$	$\therefore$	$P(A \supset B) \in [x, 1]$	(prob. informative)
$P(\neg A) = x$	$\therefore$	$P(A \wedge B) \in [0, 1 - x]$	(prob. informative)
$P(\neg A) = x$	$\therefore$	$P(B\|A) \in [0, 1]$	(prob. non-informative)

Two paradoxes of the material conditional, $A \supset B$

Example (Paradox 2)

Not-A $\therefore$ If A, then B

Not-A $\therefore$ If A, then not-B

Premise		*Conclusion*	
$\neg A$	$\therefore$	$A \supset B$	(logically valid)
$P(\neg A) = x$	$\therefore$	$P(A \supset B) \in [x, 1]$	(prob. informative)
$P(\neg A) = x$	$\therefore$	$P(A \wedge B) \in [0, 1 - x]$	(prob. informative)
$P(\neg A) = x$	$\therefore$	$P(B\|A) \in [0, 1]$	(prob. non-informative)
$\neg A$	$\therefore$	$A \supset \neg B$	(logically valid)
$P(\neg A) = x$	$\therefore$	$P(A \supset \neg B) \in [x, 1]$	(prob. informative)
$P(\neg A) = x$	$\therefore$	$P(A \wedge \neg B) \in [0, 1 - x]$	(prob. informative)
$P(\neg A) = x$	$\therefore$	$P(\neg B\|A) \in [0, 1]$	(prob. non-informative)

Experimental results

Paradox 1: $B \quad \therefore \quad$ If A, then B

Simon works in a factory that produces playing cards. He is responsible for what is printed on the cards.
On each card, there is a shape (triangle, square, . . .) of a certain color (green, blue, . . .), like:

- green triangle, green square, green circle, . . .
- blue triangle, blue square, . . .
- red triangle, . . .

Paradox 1: $B \quad \therefore \quad$ If A, then B

Simon works in a factory that produces playing cards. He is responsible for what is printed on the cards.
On each card, there is a shape (triangle, square, . . .) of a certain color (green, blue, . . .), like:

- green triangle, green square, green circle, . . .
- blue triangle, blue square, . . .
- red triangle, . . .

Imagine that a card got stuck in the printing machine. Simon cannot see what is printed on this card. Since Simon did observe the card production during the whole day, he is

[A] Pretty sure: There is a square on this card.

Considering [A], how certain can Simon be that the following sentence is true?

If there is a red shape on this card, then there is a square on this card.

Paradox 1: $B \quad \therefore \quad$ If A, then B

A Pretty sure: There is a **square** on this card.

Considering A, how certain can Simon be that the following sentence is true?

If there is a **red** shape on this card, then there is a **square** on this card.

Considering A, can Simon infer—at all—how certain he can be, that the sentence in the box is true?

- ☐ NO, Simon cannot infer his certainty.
- ☐ YES, Simon can infer his certainty.

Paradox 1: $B \quad \therefore \quad$ If A, then B

A Pretty sure: There is a **square** on this card.

Considering A, *how certain can Simon be that the following sentence is true?*

If there is a **red** shape on this card, then there is a **square** on this card.

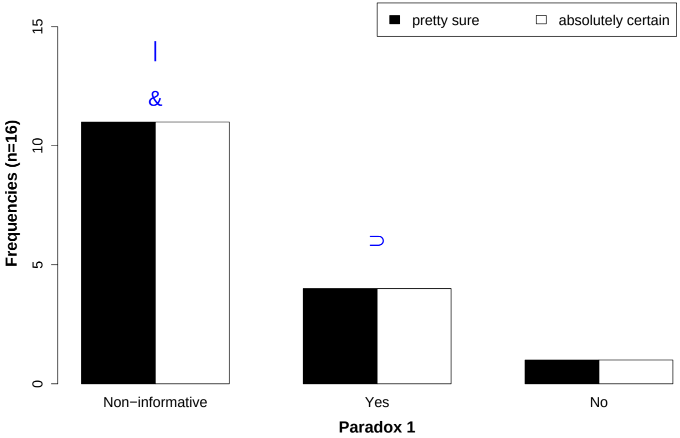
Considering A, can Simon infer—at all—how certain he can be, that the sentence in the box is true?

- ☐ NO, Simon cannot infer his certainty.
- ☐ YES, Simon can infer his certainty.

In case you ticked YES, *please fill in*

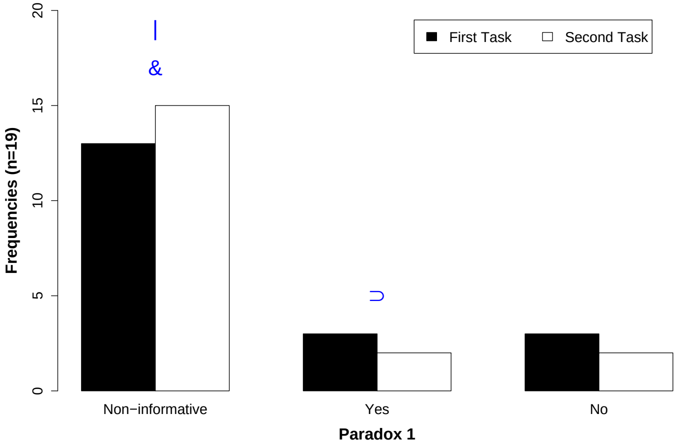
- ☐ Simon can be pretty sure that the sentence in the box is false.
- ☐ Simon can be pretty sure that the sentence in the box is true.

Paradox 1 ($n_1 = 16$)



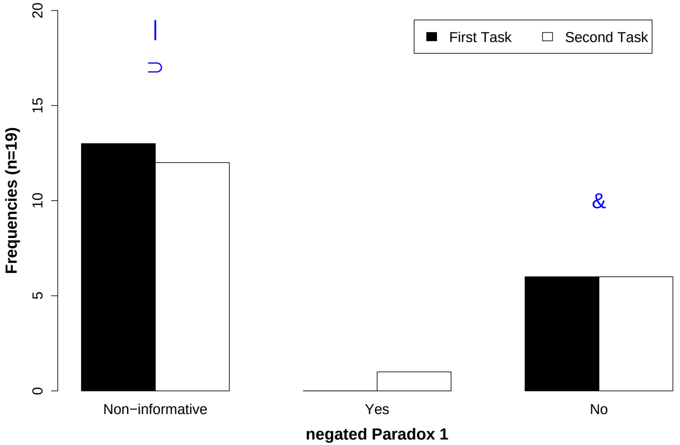
$\blacksquare \quad \& \quad \square : \quad B \quad \therefore \quad A \rightarrow B$

Paradox 1 ($n_3 = 19$)



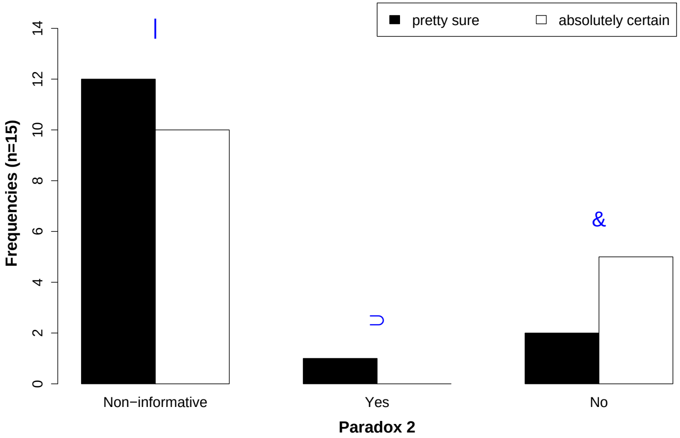
$$\blacksquare \quad \& \quad \square : \quad B \quad \therefore \quad A \rightarrow B$$

negated Paradox 1 ($n_3 = 19$)



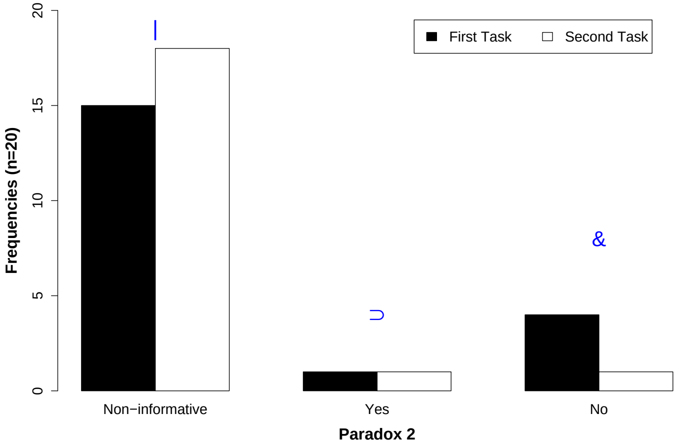
■ & □ : $B \quad \therefore \quad A \rightarrow \neg B$

Paradox 2 ($n_2 = 15$)

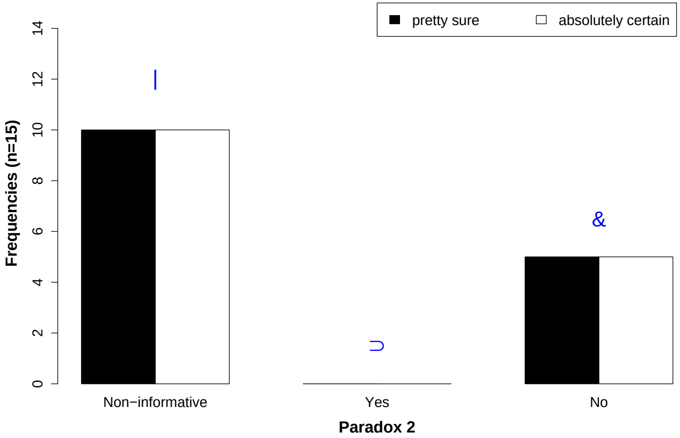


$$\blacksquare \quad \& \quad \square : \quad \neg A \quad \therefore \quad A \rightarrow B$$

Paradox 2 ($n_4 = 20$)

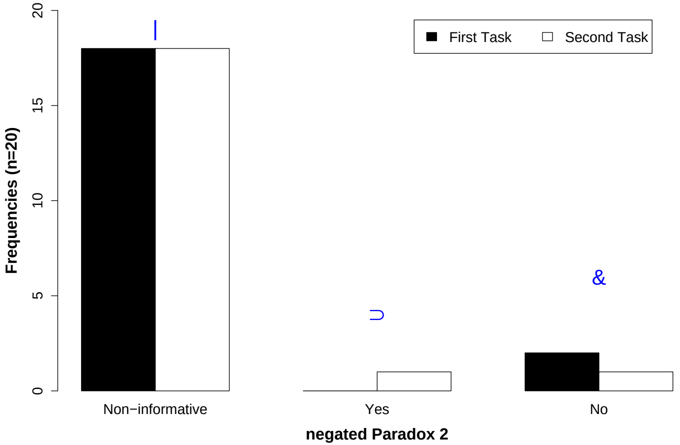


negated Paradox 2 ($n_2 = 15$)



$$\blacksquare \quad \& \quad \square : \quad \neg A \quad \therefore \quad A \rightarrow \neg B$$

negated Paradox 2 ($n_4 = 20$)



■ & □ : $\neg A \quad \therefore \quad A \to \neg B$

Complement

If A, then B $\quad\therefore\quad$ If A, then $\neg B$

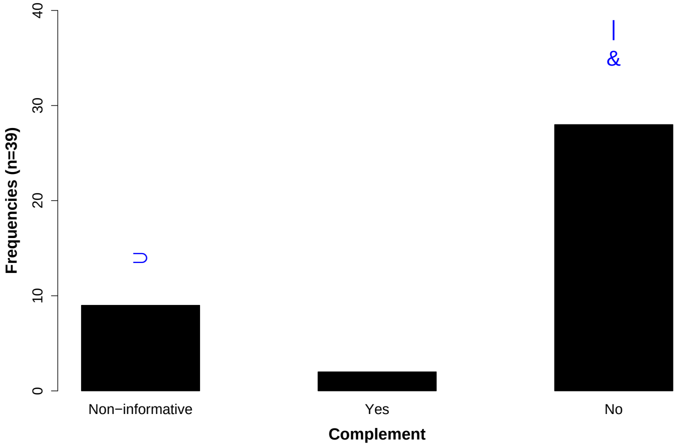
Premise		Conclusion	
$A \supset B$	$\therefore$	$A \supset \neg B$	(not logically valid)
$P(A \supset B) = x$	$\therefore$	$P(A \supset \neg B) \in [1 - x, 1]$	(prob. informative)
$P(A \wedge B) = x$	$\therefore$	$P(A \wedge \neg B) \in [0, 1 - x]$	(prob. informative)
$P(B \mid A) = x$	$\therefore$	$P(\neg B \mid A) = 1 - x$	(prob. informative)

Complement

If A, then B $\therefore$ If A, then $\neg B$

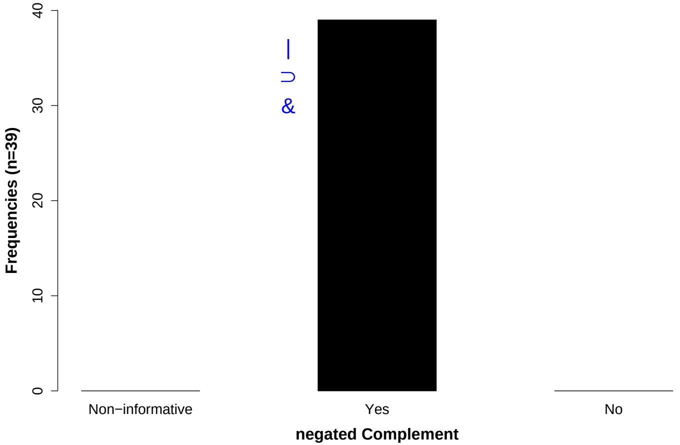
Premise		*Conclusion*	
$A \supset B$	$\therefore$	$A \supset \neg B$	(not logically valid)
$P(A \supset B) = x$	$\therefore$	$P(A \supset \neg B) \in [1 - x, 1]$	(prob. informative)
$P(A \wedge B) = x$	$\therefore$	$P(A \wedge \neg B) \in [0, 1 - x]$	(prob. informative)
$P(B\|A) = x$	$\therefore$	$P(\neg B\|A) = 1 - x$	(prob. informative)
$A \supset B$	$\therefore$	$A \supset \neg B$	(not logically valid)
$P(A \supset B) = .99$	$\therefore$	$P(A \supset \neg B) \in [.01, 1]$	(pract. non-inform.)
$P(A \wedge B) = .99$	$\therefore$	$P(A \wedge \neg B) \in [0, .01]$	(prob. informative)
$P(B\|A) = .99$	$\therefore$	$P(\neg B\|A) = .01$	(prob. informative)

Complement ($n_3 + n_4 = 39$)



$$A \rightarrow B \quad \therefore \quad A \rightarrow \neg B$$

negated Complement ($n_3 + n_4 = 39$)



$$A \rightarrow B \quad \therefore \quad A \rightarrow B$$

Paradox 3: Monotonicity (Premise strengthening)

"**If** A, **then** B" interpreted as "$A \supset B$"

$\mathfrak{P}_1$ **If** the animal is a bird, **then** it can fly

log. valid

$\mathfrak{C}$ **If** the animal is a bird **and** a penguin, **then** it can fly

$$\boxed{A \supset B} \vdash \boxed{A \wedge C \supset B}$$

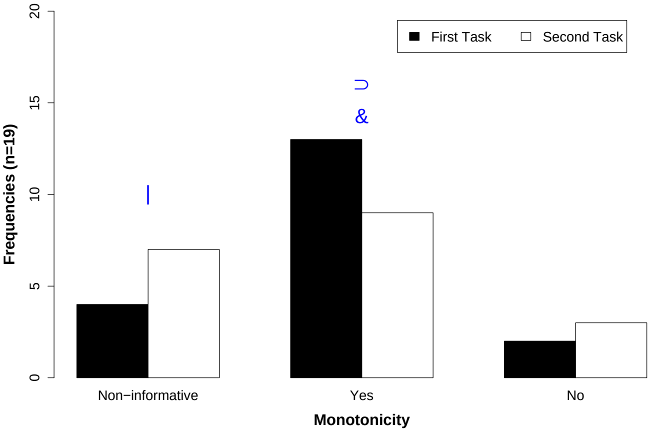
Cautious Monotonicity

"**If** A, **then** B" interpreted as "$A \supset B$"

$\mathfrak{P}_1$	**If** the animal is a bird, **then** it can fly	
$\mathfrak{P}_2$	**If** the animal is a bird, **then** it is a penguin	
		log. valid
$\mathfrak{C}$	**If** the animal is a bird **and** a penguin, **then** it can fly	

The second premise "blocks" the conclusion

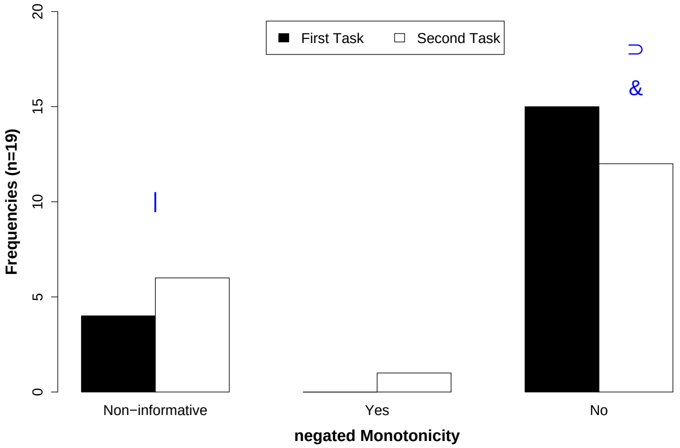
Monotonicity ($n_3 = 19$)



$\blacksquare : A \rightarrow B \quad \therefore \quad C \wedge A \rightarrow B$

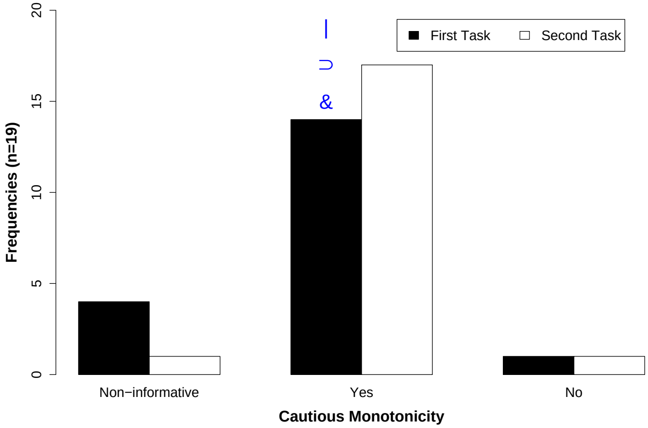
$\square : A \rightarrow B \quad \therefore \quad A \wedge C \rightarrow B$

negated Monotonicity ($n_3 = 19$)



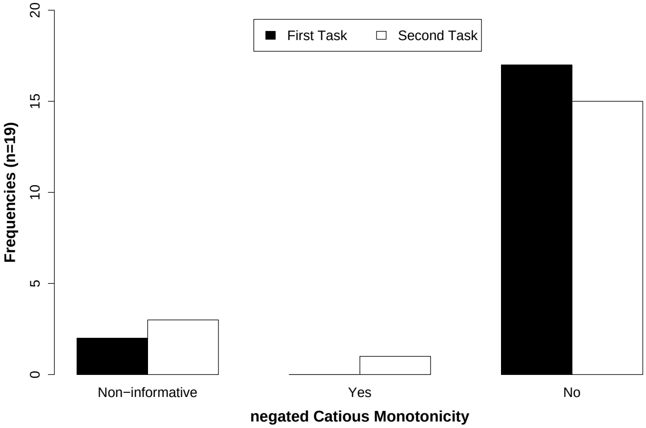
$$■ : A \rightarrow B \quad \therefore \quad C \wedge A \rightarrow \neg B$$
$$□ : A \rightarrow B \quad \therefore \quad A \wedge C \rightarrow \neg B$$

Cautious Monotonicity ($n_3 = 19$)



$$\blacksquare : A \rightarrow B, A \rightarrow C \quad \therefore \quad A \wedge C \rightarrow B$$
$$\square : A \rightarrow C, A \rightarrow B \quad \therefore \quad A \wedge C \rightarrow B$$

negated Cautious Monotonicity ($n_3 = 19$)



$$\blacksquare : A \to B, A \to C \quad \therefore \quad A \wedge C \to \neg B$$

$$\square : A \to C, A \to B \quad \therefore \quad A \wedge C \to \neg B$$

Conclusions

- Framing human inference in coherence based probability logic
  - new predictions (probabilistic (non-)informativeness)
  - new experimental paradigms
  - incomplete probabilistic knowledge leads to probability-intervals
  - investigating argument forms that differentiate

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  - investigating argument forms that differentiate
- Most participants interpret conditionals as conditional events, but...
- ...differences in interpretations may indicate intra- and interindividual differences
- Alternative interpretations, beyond $\cdot|\cdot$, $\cdot \supset \cdot$, and $\cdot \wedge \cdot$?

Acknowledgments

- EUROCORES programme LogICCC "The Logic of Causal and Probabilistic Reasoning in Uncertain Environments" (European Science Foundation)
- FWF project "Mental probability logic" (Austrian Research Fonds)

Papers to download:

`www.users.sbg.ac.at/~pfeifern/`

Appendix

Design Experiment 1

- **Two conditions**: Group 1 ($n_1 = 16$) and Group 2 ($n_2 = 15$)
- **Tasks**: Each group 20 tasks (10 arguments affirmative & negated)
- **Group 1**: Five Modus Ponens tasks and five Paradox 1 tasks with varying uncertainties of the categorical premises ("pretty sure" / "absolutely certain",e.g.);

  Modus Ponens: from $\boxed{If\ \ A,\ then\ B}$ and $\boxed{A}$ infer $\boxed{B}$

  Paradox 1: from $\boxed{B}$ infer $\boxed{If A,\ then B}$

- **Group 2**: Five Modus Ponens tasks and five Paradox 2 tasks with varying uncertainties of the categorical premises ("pretty sure" / "absolutely certain",e.g.);

  Modus Ponens: from $\boxed{If A,\ then B}$ and $\boxed{A}$ infer $\boxed{B}$

  Paradox 2: from $\boxed{\neg A}$ infer $\boxed{If A,\ then B}$

Design Experiment 2

- **Two conditions**: Group 1 ($n_3 = 19$) and Group 2 ($n_4 = 20$)
- **Tasks**: Each group 20 tasks (affirmative & negated)

Group 1	informative	not informative
	COMPLEMENT	IRRELEVANCE
	CAUT. MONOTONICITY I/II	MONOTONICITY I/II
	MODUS PONENS I/II	PARADOX 1 I/II

Group 2	informative	not informative
	COMPLEMENT	IRRELEVANCE
	MODUS TOLLENS I/II	CONTRAPOS. I/II
	dwr MONOTONICITY I/II	PARADOX 2 I/II

System P: Rationality postulates for nonmonotonic reasoning (Kraus, Lehmann & Magidor, 1990)

Reflexivity (axiom): $\alpha \mid\!\sim \alpha$

Left logical equivalence:

from $\models \alpha \equiv \beta$ and $\alpha \mid\!\sim \gamma$ infer $\beta \mid\!\sim \gamma$

Right weakening:

from $\models \alpha \supset \beta$ and $\gamma \mid\!\sim \alpha$ infer $\gamma \mid\!\sim \beta$

Or: from $\alpha \mid\!\sim \gamma$ and $\beta \mid\!\sim \gamma$ infer $\alpha \vee \beta \mid\!\sim \gamma$

Cut: from $\alpha \wedge \beta \mid\!\sim \gamma$ and $\alpha \mid\!\sim \beta$ infer $\alpha \mid\!\sim \gamma$

Cautious monotonicity:

from $\alpha \mid\!\sim \beta$ and $\alpha \mid\!\sim \gamma$ infer $\alpha \wedge \beta \mid\!\sim \gamma$

And (derived rule): from $\alpha \mid\!\sim \beta$ and $\alpha \mid\!\sim \gamma$ infer $\alpha \mid\!\sim \beta \wedge \gamma$

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$\alpha \mid\!\sim \beta$	is read as	If α, $\underbrace{\textit{normally}}_{?}$ β

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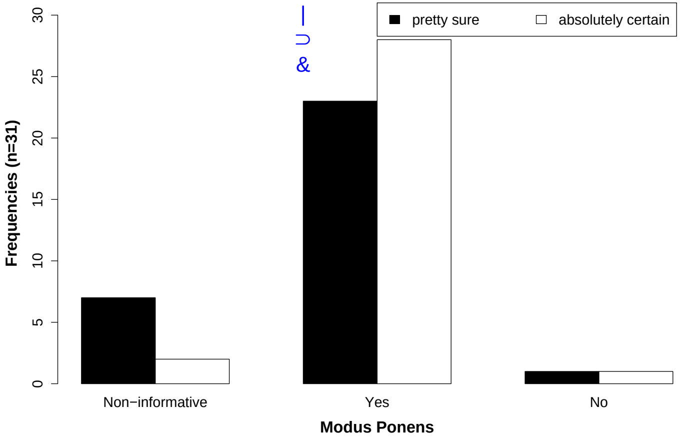
Semantics for System P

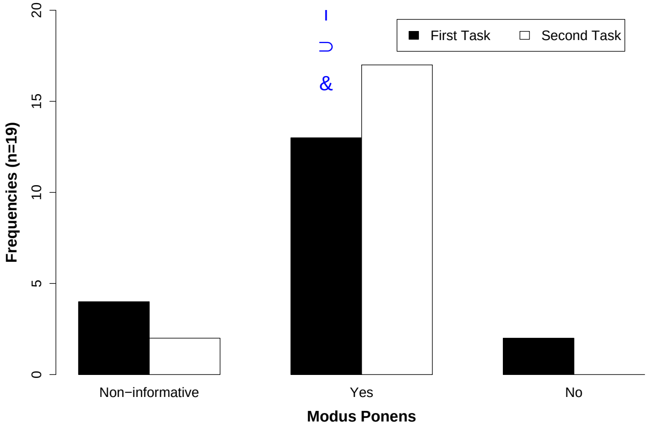
- Normal world semantics (Kraus, Lehmann & Magidor '90)
- Possibility semantics: $\alpha \mid\!\sim \beta$ iff $\Pi(A \wedge B) > \Pi(A \wedge \neg B)$ (e.g., Benferhat, Dubois & Prade '97)
  - Empirical support: Da Silva Neves, Bonnefon, & Raufaste ('02), Benferhat, Bonnefon, Da Silva Neves ('05)
- Inhibition nets (Leitgeb '01, '04)
- Probability semantics
  - Infinitesimal: $\alpha \mid\!\sim \beta$ iff $P(\beta|\alpha) = 1 - \epsilon$ (e.g., Adams '75)
  - Noninfinitesimal: $\alpha \mid\!\sim \beta$ iff $P(\beta|\alpha) > .5$ (e.g., Gilio '02; Biazzo, Gilio, Lukasiewicz, Sanfilippo, '05)
  - ...
    - Empirical support: Pfeifer & Kleiter ('03, '05, '06)

Modus Ponens $(n_1 + n_2 = 31)$



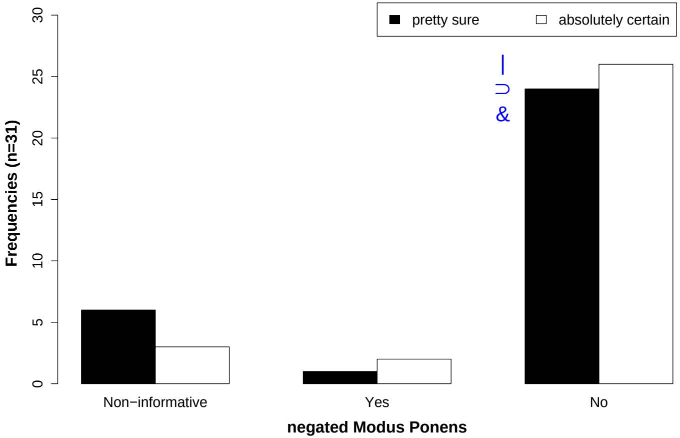
$$\blacksquare \quad \& \quad \square : \quad A \rightarrow B, \quad A \quad \therefore \quad B$$

Modus Ponens ($n_3 = 19$)



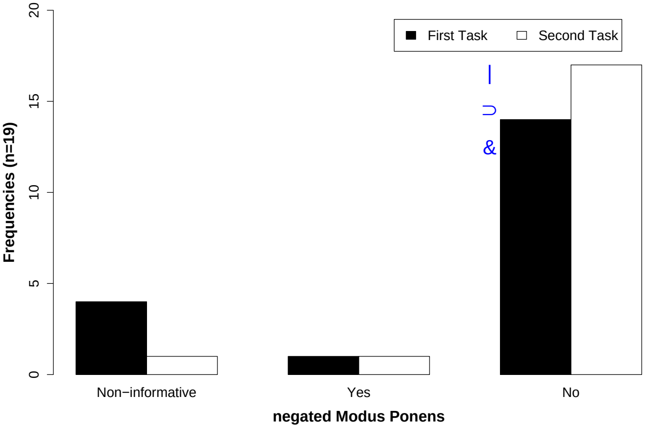
$$\blacksquare : A \rightarrow B, A \quad \therefore \quad B$$
$$\square : A, A \rightarrow B \quad \therefore \quad B$$

negated Modus Ponens ($n_1 + n_2 = 31$)



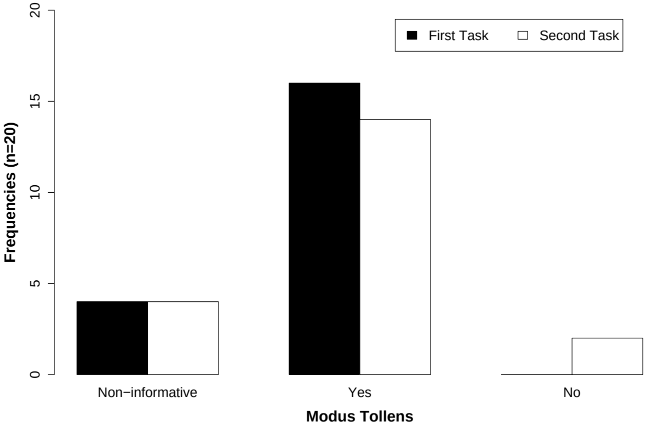
$$\blacksquare \quad \& \quad \square : \quad A \to B, \quad A \quad \therefore \quad \neg B$$

negated Modus Ponens ($n_3 = 19$)



$$\blacksquare : A \rightarrow B, A \quad \therefore \quad \neg B$$
$$\square : A, A \rightarrow B \quad \therefore \quad \neg B$$

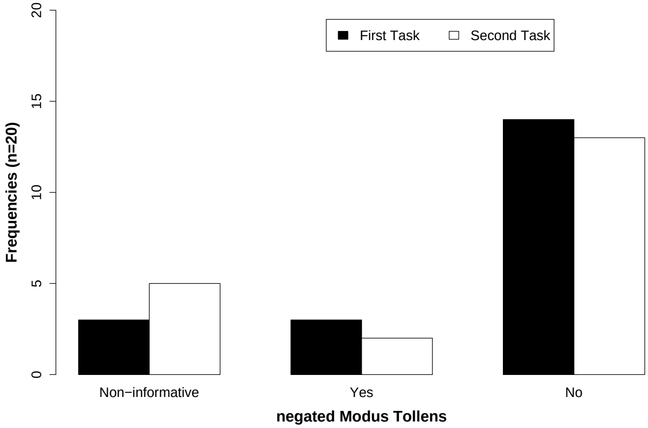
Modus Tollens ($n_4 = 20$)



$\blacksquare : \neg B, A \rightarrow B \quad \therefore \quad \neg A$

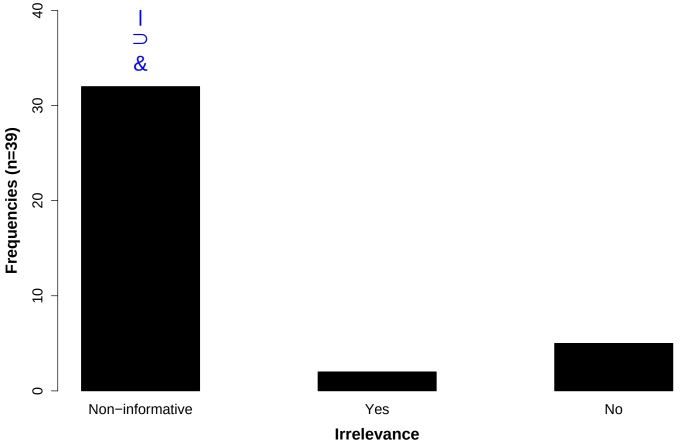
$\square : A \rightarrow B, \neg B \quad \therefore \quad \neg A$

negated Modus Tollens ($n_4 = 20$)



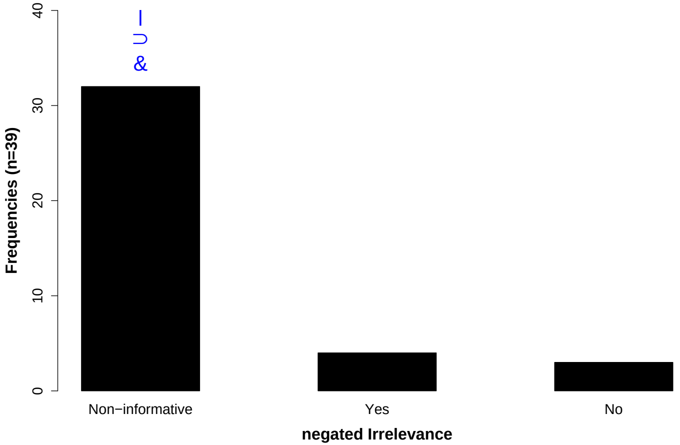
$$\blacksquare : \neg B, A \rightarrow B \quad \therefore \quad A$$
$$\square : A \rightarrow B, \neg B \quad \therefore \quad A$$

Irrelevance ($n_3 + n_4 = 39$)



$$A \rightarrow B \quad \therefore \quad A \rightarrow C$$

negated Irrelevance ($n_3 + n_4 = 39$)



$$A \to B \quad \therefore \quad A \to \neg C$$