A Probabilistic Semantics for Counterfactuals

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- If it were the case that A, then it would be the case that B

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Plan of the talk:

1. A New Semantics: The Popper Function Semantics
2. Interpreting the Semantics
3. What Becomes of the Centering Axioms?
4. An Equivalent Semantics: The Probabilistic Limit Semantics
5. An Application: Are Most (Ordinary) Counterfactuals False?

(I have just finished a draft on this.)

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Strategy: Let the truth of, e.g., *'If the match were struck, it would light'* consist in its corresponding conditional probability being high.

As a prerequisite, our new semantics will involve quantification over conditional probability functions (see Popper 1959, Stalnaker 1970, and others).

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As a prerequisite, our new semantics will involve quantification over conditional probability functions (see Popper 1959, Stalnaker 1970, and others).

As we will show later, these conditional probability functions allow for interpreting '$\mathfrak{P}(Y|X) = 1$' as stating

- the probability of Y given X is high, i.e., close to 1

where 'high' and 'close' are vague terms.

Let $\mathfrak{A}$ be a Boolean field on W.

Definition

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2. $\mathfrak{P}(X|X) = 1$,
3. If $\mathfrak{P}(W \backslash X \,|\, X) \neq 1$ then $\mathfrak{P}(.|X)$ is a (finitely additive) probability measure,
4. Multiplication Axiom: $\mathfrak{P}(X \cap Y|Z) = \mathfrak{P}(X|Z)\,\mathfrak{P}(Y|X \cap Z)$,
5. If $\mathfrak{P}(X|Y) = \mathfrak{P}(Y|X) = 1$, then for all $Z \in \mathfrak{A}$: $\mathfrak{P}(Z|X) = \mathfrak{P}(Z|Y)$.

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Absolute probabilities may be introduced by means of:

$$P(X) = \mathfrak{P}(X|W)$$

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From 4 it follows: If the "absolute probability" $\mathfrak{P}(X|W) > 0$, then

$$\text{Ratio Formula: } \frac{\mathfrak{P}(X \cap Y|W)}{\mathfrak{P}(X|W)} = \mathfrak{P}(Y|X \cap W) = \mathfrak{P}(Y|X)$$

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Conditional probability measures are to be considered conceptually primitive, rather than being reducible to absolute probability (cf. Hájek 2003).

This is thus our new probabilistic semantics for counterfactuals:

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  - A new probabilistic truth condition for subjunctive conditionals:

$$w \in [\![A \,\Box\!\!\rightarrow B]\!] \text{ if and only if } \mathfrak{P}_w([\![B]\!] | [\![A]\!]) = 1$$

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A formula A in $\mathcal{L}$ is logically true in the Popper function semantics iff A is true in every world in every Popper function model.

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Theorem

The system V of conditional logic is sound and complete with respect to the Popper function semantics for subjunctive conditionals.

- Rules of V:
  1. Modus Ponens (for $\supset$)
  2. Deduction within subjunctive conditionals: for any $n \geq 1$

$$\frac{\vdash (B_1 \wedge \ldots \wedge B_n) \supset C}{\vdash ((A \,\Box\!\!\rightarrow B_1) \wedge \ldots \wedge (A \,\Box\!\!\rightarrow B_n)) \supset (A \,\Box\!\!\rightarrow C)}$$

  3. Interchange of logical equivalents
- Axioms of V:
  1. Truth-functional tautologies
  2. $A \,\Box\!\!\rightarrow A$
  3. $(\neg A \,\Box\!\!\rightarrow A) \supset (B \,\Box\!\!\rightarrow A)$
  4. $(A \,\Box\!\!\rightarrow \neg B) \vee (((A \wedge B) \,\Box\!\!\rightarrow C) \leftrightarrow (A \,\Box\!\!\rightarrow (B \supset C)))$

Interpreting the Semantics

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- In the semantics, we demand

$$w \in [\![A \mathrel{\Box\!\!\rightarrow} B]\!] \text{ if and only if } \mathfrak{P}_w([\![B]\!]|[\![A]\!]) = 1$$

but *not*

$$\mathfrak{P}_w([\![A \mathrel{\Box\!\!\rightarrow} B]\!]|W) = \mathfrak{P}_w([\![B]\!]|[\![A]\!])$$

(hence the semantics does not run into Lewis' Triviality result).

But we still know such a semantics is problematic?
(cf. Edgington 1995 & 2008, Bennett 2003):

- *“Prima facie, there is room for an account of objectively correct conditional thoughts. . . ‘If A, B’ is true iff the objective probability of B given A is sufficiently high. This is not compatible with the Thesis, and is independently objectionable. (I do not object to the fact that the truth condition is vague.)”* (Edgington 1995, p.292)

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- However, the "objections" vanish as long as one is willing to distinguish:

$$(1a)\quad \mathfrak{Cr}([\![B]\!]|[\![A]\!]) = \mathfrak{Cr}([\![A \mathrel{\Box\!\!\rightarrow} B]\!] \mid W)$$

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- Moreover, *"The truth condition has the additional embarrassing consequence that the truth of 'If A, B' is compatible with the truth of $A\&\neg B$"*: $\hookrightarrow$ But that's exactly what we want!

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- Secondly, we allow for deviations from the truth conditions of our everyday $\Box\!\!\rightarrow$ if this leads to a better theory, i.e., if this avoids philosophical problems and makes the semantics continuous with science.

  Such deviations might even be *necessary* if our common sense $\Box\!\!\rightarrow$ does not have a reference or if our standard semantical theory misdescribes what it refers to.

What Becomes of the Centering Axioms?

If compared to Lewis' system VC, the only missing logical axioms are:

C1 Weak Centering: $(A \mathrel{\Box\!\!\rightarrow} B) \supset (A \supset B)$

C2 Centering: $(A \wedge B) \supset (A \mathrel{\Box\!\!\rightarrow} B)$

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Why do they fail?

$\mathfrak{P}_w([\![B]\!] | [\![A]\!])$ is not necessarily tied to a particular distribution of truth values of A and B in w!

Is this a problem? Not necessarily.

$A \wedge B$ should not make $A \,\Box\!\!\rightarrow B$ true, nor should $A \wedge \neg B$ make $A \,\Box\!\!\rightarrow B$ false(?).

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Contra Weak Centering: w might be exceptional with respect to its own probabilistic standards.

Pro Weak Centering: It entails *counterfactual MP/MT* to be valid.

It is possible to restore Centering by imposing extra constraints:

Actual Determinism corresponds to *Centering*:

- Semantic constraint:
  For all $w \in W$, for all $A \in \mathcal{L}$: $\mathfrak{P}_w([\![A]\!] \,|\, W)$ equals the truth value of A in w (hence $\mathfrak{P}_w([\![A]\!] \,|\, W)$ only takes values in $\{0,1\}$).
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Counterfactual Determinism corresponds to *Conditional Excluded Middle* (Stalnaker's axiom):

- Semantic constraint:
  For all $w \in W$: $\mathfrak{P}_w(.|.)$ only takes values in $\{0,1\}$.
- Characteristic axiom:
  $(A \,\Box\!\!\rightarrow B) \vee (A \,\Box\!\!\rightarrow \neg B)$

One can also find "approximations" of the Centering axioms without adding constraints on our models:

- The following similar-looking axioms *are* logically true:

  $(A \mathrel{\Box\!\!\rightarrow} B) \supset (\top \mathrel{\Box\!\!\rightarrow} (A \supset B))$
  $(\top \mathrel{\Box\!\!\rightarrow} (A \wedge B)) \supset (A \mathrel{\Box\!\!\rightarrow} B)$

Or one "saves" counterfactual MP (MT) *pragmatically* as follows (omitting '$[\![.]\!]$'):

- By a variant of the Principal Principle,

$$\mathfrak{Cr}(B|A \wedge P(B|A) = r \wedge C) = r$$

for admissible C, an "initial" credence function Cr, and P expressing $\mathfrak{P}$.

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- I.e., if '$\rightarrow$' is the indicative 'if-then', then by Ernest Adams' semantics

$$A \wedge (A \,\Box\!\!\rightarrow B) \rightarrow B$$

  gets assigned a conditional subjective probability of 1 by all credence functions $\mathfrak{Cr}'$ that are sufficiently like the "initial" $\mathfrak{Cr}$.

  So if A and $A \,\Box\!\!\rightarrow B$ are assertable according to $\mathfrak{Cr}'$, then B is so as well.

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$\langle W, \mathfrak{A}, ((P_i^w)_{i \in I_w})_{w \in W}, (\leq^w)_{w \in W}, [\![.]\!] \rangle$ is a *probabilistic limit model* for subjunctive conditionals iff

- W is a non-empty set of possible worlds,
- $\mathfrak{A} = \{[\![A]\!] \,|\, A \in \mathcal{L}\}$, for $\mathcal{L}$ as before,

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In order to interpret the Popper function semantics properly, it is illuminating to compare it with an alternative probabilistic semantics:

$\langle W, \mathfrak{A}, ((P_i^w)_{i \in I_w})_{w \in W}, (\leq^w)_{w \in W}, [\![.]\!] \rangle$ is a *probabilistic limit model* for subjunctive conditionals iff

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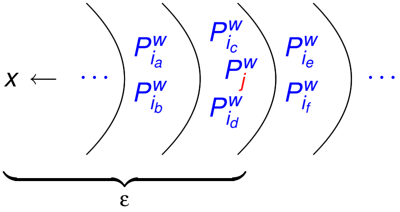
- the *Convergence Assumption* is satisfied:

  for all $w \in W$ and $A, B \in \mathcal{L}$, either there is no $i \in I_w$ such that $P_i^w([\![A]\!]) > 0$, or the sequence $\left(\frac{P_i^w([\![B \wedge A]\!])}{P_i^w([\![A]\!])}\right)_{i \in I_w}$ converges.

- $\left(\frac{P_i^w(\llbracket B \wedge A \rrbracket)}{P_i^w(\llbracket A \rrbracket)}\right)_{i \in I_w}$ is said to converge to $x \in [0,1]$ if and only if for all $\varepsilon > 0$ there is an index $j \in I_w$ with $P_j^w(\llbracket A \rrbracket) > 0$, such that for all $i \leq^w j$ with $P_i^w(\llbracket A \rrbracket) > 0$ it holds that $\left|\frac{P_i^w(\llbracket B \wedge A \rrbracket)}{P_i^w(\llbracket A \rrbracket)} - x\right| < \varepsilon$.



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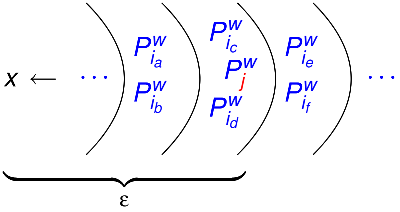

- $\llbracket . \rrbracket : \mathcal{L} \to \wp(W)$ satisfies the following semantic rules:
  - Standard semantic rules for classical propositional connectives.
  - $w \in \llbracket A \mathrel{\Box\!\!\rightarrow} B \rrbracket$ if and only if either of the following is satisfied:
    - There is no $i \in I_w$, such that $P_i^w(\llbracket A \rrbracket) > 0$.
    - It holds that:

$$\lim_{i \in I_w}\left(\frac{P_i^w(\llbracket B \wedge A \rrbracket)}{P_i^w(\llbracket A \rrbracket)}\right) = 1$$

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Theorem

- *Every family $(P_i)_{i \in I}$ of probability measures (on the same countable algebra $\mathfrak{A}$) which satisfies the Convergence Assumption with respect to a linear preorder $\leq$, represents a Popper function $\mathfrak{P}$ (on $\mathfrak{A}$), where the representation is given by:*

Repr *If there is an $i \in I$, such that $P_i(X) > 0$, then*

$$\mathfrak{P}(Y|X) = \lim_{i \in I}\left(\frac{P_i(Y \cap X)}{P_i(X)}\right)$$

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(This is some improvement on van Fraassen 1976 and related results.)

- Hence, the Popper function semantics can also be regarded based on *comparative similarity*, but now similarity of *absolute probability functions*:
  - $\mathfrak{P}_w(.|W)$ may be interpreted as the *actual* absolute probability measure.
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(Alternative: Lehmann&Magidor 1992, McGee 1994, Halpern 2001 on *non-standard* P)

Perhaps scientifically supported Popper functions can be used to clarify the metaphysics of a (quasi-)Lewisian semantics for counterfactuals?

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According to our probabilistic semantics, the latter counterfactual might still be true as long as exceptions to it have a probability close to 0.

(This is not quite the end of the story…)